Homework I 2018-2019 Spring MECE 588

(Q1)

$$x' = \begin{bmatrix} 0 & -2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2 \\ 0 & 0 & 2 & 0 \end{bmatrix} x + \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} u \qquad y = [e\ f\ g\ h]\, x$$

(a) char-equation $= d(s) = (s+2j)^2 (s-2j)^2$ $\quad s_{1,2} = \mp j$

minimal-polynomial $= m(s) = (s+2j)(s-2j)$

(a) stable in the sense of Lyapunov ; ~~since the~~
The $d(s)$ "characteristic polynomial" has zeros whose real parts are equal to "0" however at the minimal polynomial the roots that having zero real parts have multiplicity "1" in the minimal polynomial

(b) Not asymptotically stable as there are roots of $d(s)$ which are at the imaginary axis

(c) (i) All eigenvalues should have non-positive real parts ✓ true

~~$s_{1,2,3,4} = \mp j$~~ $\quad s_1 = 2j,\ s_2 = -2j,\ s_3 = 2j,\ s_4 = -2j$

(ii) All eigenvalues with zero real parts are simple zeros of minimal polynomial ✓ true

(iii) $a, b, c, d = 0$ to guarantee that along the invariant subspaces ~~of~~ associated with eigenvalues of A with zero real part ($s_1 = 2j,\ s_2 = -2j,\ s_3 = 2j,\ s_4 = -2j$) should ~~have~~ ~~zero~~ correspond to B vector with zero component

Hence $B = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ ✓

(d) The transfer function $T(s) = \dfrac{N(s)}{(s+2j)(s-2j)} = \dfrac{N(s)}{(s^2+4)}$

Q1 (d) continue

This transfer function should have $N(s) = 0$ to have BIBO stability condition as the poles $s_1 = +j$ $s_2 = -j$ are not at open LHP (they are over imaginary axis)

$$T(s) = (e\ f\ g\ h)\,(sI - A)^{-1}\begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}$$

$$T(s) = (e\ f\ g\ h)\begin{bmatrix} s & 2 & 0 & 0 \\ -2 & s & 0 & 0 \\ 0 & 0 & s & 2 \\ 0 & 0 & -2 & s \end{bmatrix}^{-1}\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

~~to guarantee this~~

$$\cancel{f\,(e\ f\ g\ h)\,\frac{\begin{bmatrix} s & -2 & 0 & 0 \\ +2 & s & 0 & 0 \\ 0 & 0 & s & -2 \\ 0 & 0 & 2 & s \end{bmatrix}}{(s^2+4)^2}\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}}$$

$$T(s) = (e\ f\ g\ h)\,\frac{\begin{bmatrix} s(s^2+4) & +2(s^2+4) & 0 & 0 \\ -2(s^2+4) & s(s^2+4) & 0 & 0 \\ 0 & 0 & s(s^2+4) & +2(s^2+4) \\ 0 & 0 & -2(s^2+4) & s(s^2+4) \end{bmatrix}^{T}}{(s^2+4)(s^2+4)}\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

$$T(s) = (e \ f \ g \ h) \frac{\begin{bmatrix} s(s^2+4) & -2(s^2+4) & 0 & 0 \\ 2(s^2+4) & s(s^2+4) & 0 & 0 \\ 0 & 0 & s(s^2+4) & -2(s^2+4) \\ 0 & 0 & 2(s^2+4) & s(s^2+4) \end{bmatrix}}{(s^2+4)^2} \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}$$

$$T(s) = (e \ f \ g \ h) \begin{bmatrix} \frac{s}{s^2+4} & \frac{-2}{s^2+4} & 0 & 0 \\ \frac{2}{s^2+4} & \frac{s}{s^2+4} & 0 & 0 \\ 0 & 0 & \frac{s}{s^2+4} & \frac{-2}{s^2+4} \\ 0 & 0 & \frac{2}{s^2+4} & \frac{s}{s^2+4} \end{bmatrix} \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}$$

$$T(s) = (e \ f \ g \ h) \begin{pmatrix} \frac{as-2b}{s^2+4} \\ \frac{2a+bs}{s^2+4} \\ \frac{cs-2d}{s^2+4} \\ \frac{2c+ds}{s^2+4} \end{pmatrix}$$

$$T(s) = e\left(\frac{as-2b}{s^2+4}\right) + f\left(\frac{2a+bs}{s^2+4}\right) + g\left(\frac{cs-2d}{s^2+4}\right) + h\left(\frac{2c+ds}{s^2+4}\right) = \frac{N(s)}{s^2+4}$$

$N(s)=0$ only iff $a=b=c=d$ or $e=f=g=h=0$ //

Homework II

2018-2019 Spring MECE 548

(Q1) $\dot{x} = \begin{bmatrix} 0 & -2 & 0 & 0 \\ 2 & 0 & 1 & 0 \\ 0 & 0 & 0 & -2 \\ 0 & 0 & 2 & 0 \end{bmatrix} x + \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} u$ $y = [e \ f \ g \ h] x$

$d(s) = (s+2j)^2 (s-2j)^2 = (s^2+4)^2$ characteristic polynomial

$m(s) = (s+2j)^2 (s-2j)^2 = (s^2+4)^2$ minimal polynomial

(a) Not stable in the sense of Lyapunov as minimal polynomial has root whose real parts is equal to "0", and the multiplicity of these roots in the minimal polynomial is greater than "1"

(b) Not asymptotically stable as not stable in the sense of Lyapunov

(c) (i) All eigenvalues have non-positive real parts ✓

(ii) All eigenvalues with zero real parts are simple zeros of minimal polynomial (wrong) ✗

as the $m(s) = (s+2j)^2 (s-2j)^2$

No need to check condition (iii)

Not BIBS stable

$$T(s) = \frac{N(s)}{(s^2+4)^2}$$

This transfer function should have $N(s)=0$ to have BIBO stability as the poles $s_{1,2} = +j$ $s_{3,4} = -j$ are not at ~~at~~ open LHP

$$T(s) = (e\ f\ g\ h)\ (sI-A)^{-1}\begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = (e\ f\ g\ h)\begin{bmatrix} s & 2 & 0 & 0 \\ -2 & s & 0 & 0 \\ 0 & 0 & s & 2 \\ 0 & 0 & -2 & s \end{bmatrix}\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

$$T(s) = (e\ f\ g\ h)\frac{\begin{bmatrix} s(s^2+4) & 2(s^2+4) & 0 & 0 \\ -2(s^2+4) & s(s^2+4) & 0 & 0 \\ -2s & -s^2 & s(s^2+4) & 2(s^2+4) \\ 4 & 2s & -2(s^2+4) & s(s^2+4) \end{bmatrix}^T}{(s^2+4)^2}\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

$$= (e\ f\ g\ h)\frac{\begin{bmatrix} s(s^2+4) & -2(s^2+4) & -2s & 4 \\ 2(s^2+4) & s(s^2+4) & -s^2 & 2s \\ 0 & 0 & s(s^2+4) & -2(s^2+4) \\ 0 & 0 & 2(s^2+4) & s(s^2+4) \end{bmatrix}}{(s^2+4)^2}\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

$T(s)=0$ if and only if $a=b=c=d=e=f=g=h=0$

Homework 3 2018-2019 spring MECE 568

Q1 $\dot{x} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -2 \end{bmatrix} x + \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} u \qquad y = [e \; f \; g \; h] x$

$d(s) = s^2(s+1)(s+2)$
$m(s) = s^2(s+1)(s+2) \rightarrow$ minimal polynomial

(a) Not stable in the sense of Lyapunov as $s=0$ is a root of $m(s)$ with multiplicity greater than 1 (its multiplicity in $m(s)$ is 2.

(b) Not asymptotically stable as it is not Lyapunov stable

(c) (i) All eigenvalues should have positive real parts. ✓ true
$s_{1,2}=0 \quad s_3=-1 \quad s_4=-2$

(ii) All eigenvalues with zero real parts are simple zeros of minimal polynomial ✗ false

as (ii) is not satisfied ⇒ not BIBS stable

(d) $T(s) = (e \; f \; g \; h) \begin{bmatrix} s & -1 & 0 & 0 \\ 0 & s & 0 & 0 \\ 0 & 0 & s+1 & 0 \\ 0 & 0 & 0 & s+2 \end{bmatrix}^{-1} \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}$

↓
transfer function

$$T(s) = (e \; f \; g \; h) \frac{\begin{bmatrix} s(s+1)(s+2) & 0 & 0 & 0 \\ (s+1)(s+2) & s(s+1)(s+2) & 0 & 0 \\ 0 & 0 & s^2(s+2) & 0 \\ 0 & 0 & 0 & s^2(s+1) \end{bmatrix}^T}{s^2(s+1)(s+2)} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

$$T(s) = (e \; f \; g \; h) \begin{bmatrix} \frac{1}{s} & \frac{1}{s^2} & 0 & 0 \\ 0 & \frac{1}{s} & 0 & 0 \\ 0 & 0 & \frac{1}{s+1} & 0 \\ 0 & 0 & 0 & \frac{1}{s+2} \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

$$T(s) = (e \; f \; g \; h) \begin{pmatrix} \frac{a}{s} + \frac{b}{s^2} \\ \frac{b}{s} \\ \frac{c}{s+1} \\ \frac{d}{s+2} \end{pmatrix} = e\left(\frac{a}{s} + \frac{b}{s^2}\right) + f\frac{b}{s} + \frac{gc}{s+1} + \frac{hd}{s+2}$$

either ~~ex~~ - $e = f = 0$ or $a = b = 0$ for BIBO stability as this makes all poles at open LHP

Homework 3

Q2) $\dot{x} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -2 \end{bmatrix} x + \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} u \qquad y = (e\ f\ g\ h)\, x$

$d(s) = s^2(s+1)(s+2) \rightarrow$ char equation $\qquad s_{1,2} = 0 \quad s_3 = -1 \quad s_4 = -2$ eigenvalues

$m(s) = s(s+1)(s+2)$

a) Stable in the sense of Lyapunov as all roots of $m(s)$ are at LHP and the pole on the imaginary axis has multiplicity "1" in minimal polynomial

b) Not asymptotically stable as there is root at the imaginary axis at $d(s)$

c) (i) All eigenvalues should have non-positive real parts ✓ true

(ii) Eigenvalues with "0" real parts should have multiplicity "1" in minimal polynomial ✓

(iii) Invariant subspaces associated with eigenvalues of A with zero real parts (namely $s_1 = 0$ $s_2 = 0$) should correspond to B vector with zero components that means

$B = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ c \\ d \end{bmatrix} \qquad a = 0 \quad b = 0$ to satisfy BIBS stability

(d) For BIBO stability $T(s) = C(sI-A)^{-1}B$

$$T(s) = \begin{pmatrix} e & f & g & h \end{pmatrix} \begin{bmatrix} s & 0 & 0 & 0 \\ 0 & s & 0 & 0 \\ 0 & 0 & s+1 & 0 \\ 0 & 0 & 0 & s+2 \end{bmatrix}^{-1} \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}$$

$$= \begin{pmatrix} e & f & g & h \end{pmatrix} \begin{pmatrix} \frac{1}{s} & 0 & 0 & 0 \\ 0 & \frac{1}{s} & 0 & 0 \\ 0 & 0 & \frac{1}{s+1} & 0 \\ 0 & 0 & 0 & \frac{1}{s+2} \end{pmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

$$\begin{pmatrix} e & f & g & h \end{pmatrix} \begin{pmatrix} \frac{a}{s} \\ \frac{b}{s} \\ \frac{c}{s+1} \\ \frac{d}{s+2} \end{pmatrix} = \frac{ea}{s} + \frac{bf}{s} + \frac{gc}{s+1} + \frac{hd}{s+1} = T(s)$$

To have BIBO stability

either $e=f=0$ or $e=b=0$ or $a=b=0$ or $a=f=0$

Hence $T(s)$ will have poles all at LHP.