MECE 548 (week 10)

Observable Canonical Form

$$T(s) = \frac{Y(s)}{U(s)} = \frac{b_1 s^{n-1} + b_2 s^{n-2} + \dots + b_{n-1} s + b_n}{s^n + a_1 s^{n-1} + a_2 s^{n-2} + a_3 s^{n-3} + \dots + a_{n-1} s + a_n}$$

$$Y(s)\left(s^n + a_1 s^{n-1} + a_2 s^{n-2} + a_3 s^{n-3} + \dots + a_{n-1} s + a_n\right) = U(s)\left(b_1 s^{n-1} + b_2 s^{n-2} + \dots + b_{n-1} s + b_n\right)$$

Devide every part by s^n

$$Y(s)\left(1 + \frac{a_1}{s} + \frac{a_2}{s^2} + \dots + \frac{a_n}{s^n}\right) = \left(\frac{b_1}{s} + \frac{b_2}{s^2} + \dots + \frac{b_n}{s^n}\right) U(s)$$

$$Y(s) = \left(\frac{b_1}{s} + \frac{b_2}{s^2} + \dots + \frac{b_n}{s^n}\right) U(s) - \left(\frac{a_1}{s} + \frac{a_2}{s^2} + \dots + \frac{a_n}{s^n}\right) Y(s)$$

$$Y(s) = \frac{1}{s}\left\{b_1 U(s) - a_1 Y(s) + \frac{1}{s}\left\{b_2 U(s) - a_2 Y(s) + \frac{1}{s}\left\{ \dots + \frac{1}{s}\left\{b_n U(s) - a_n Y(s)\right\}\right\}\right\}\right\}$$

Let's define the states as follows:

$$X_1(s) = \frac{1}{s}\left\{b_n U(s) - a_n Y(s)\right\}$$

$$X_2(s) = \frac{1}{s}\left\{b_{n-1} U(s) - a_{n-1} Y(s) + X_1(s)\right\}$$

$$\vdots$$

$$X_i(s) = \frac{1}{s}\left\{b_{n-i+1} U(s) - a_{n-i+1} Y(s) + X_{i-1}(s)\right\}$$

$$\vdots$$

$$X_n(s) = \frac{1}{s}\left\{b_1 U(s) - a_1 Y(s) + X_{n-1}(s)\right\}$$