Week 13

*-To sum up-

$c \neq 0$ (for the eigenvalue $\lambda_1 = 1$)
for controllability of the corresponding modes.

$k \neq 0$ (for the eigenvalue $\lambda_2 = 2$)
for controllability of the corresponding modes.

Thus for the complete controllability of the system

$c \neq 0$, $k \neq 0$ and "a" and "b" values of the vector can be arbitrarily chosen.

(b) What condition is necessary such that system representation is completely observable?

Use Hautus test

For $\lambda_1 = 1$

$$\begin{bmatrix} \lambda_1 I - A \\ \hline C \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ d & e & f & m \end{bmatrix} \begin{matrix} \rightarrow r_1 \\ \rightarrow r_2 \\ \\ \rightarrow r_3 \\ \rightarrow r_4 \end{matrix}$$

r_1, r_2, r_3, r_4 are rows which are non-zero vectors

Week 13

As seen (r_1, r_2, r_3) are linearly independent, to have (r_1, r_2, r_3, r_4) to be lineally independant in $[d \; e \; f \; m] = r_4$ vector $d \neq 0$. So if $d \neq 0$, whatever the value of e, f and m, the rows r_1, r_2, r_3, r_4 will be linearly independent.

Hence the matrix will be rank $= 4$ and the model corresponding to eigenvalue $\lambda_1 = 1$ will all be observable.

For $\lambda_2 = 2$

$$\begin{bmatrix} \lambda_2 I - A \\ \hline C \end{bmatrix} = \begin{array}{c} \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ d & e & f & m \end{bmatrix} \end{array} \begin{array}{l} \rightarrow r_1 \\ \rightarrow r_2 \\ \rightarrow r_3 \\ \\ \rightarrow r_4 \end{array}$$

$r_1; r_2; r_3; r_4$ are rows which are non-zero vectors

As seen (r_1, r_2, r_3) are linearly independant, to have (r_1, r_2, r_3, r_4) to be linearly independant in $[d \; e \; f \; m] = r_4$ vector $m \neq 0$. So if $m \neq 0$ whatever the value of d, e and f, the rows r_1, r_2, r_3, r_4 will be linearly independent.

Hence the matrix will be rank=4 and the mode corresponding to eigenvalue $\lambda_2=2$ will be observable

* To sum-up

$d \neq 0$ (for the eigenvalue $\lambda_1=1$) for observability of the corresponding mode.

$m \neq 0$ (for the eigenvalue $\lambda_2=2$) for observability of the corresponding mode.

Thus for the complete observability of the system

$d \neq 0$, $m \neq 0$ and "e" and "f" values of the vector can be arbitrarily chosen.

Ex (3):

$$\dot{x} = \underbrace{\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}}_{A} x + \underbrace{\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}}_{B} u \qquad y = \underbrace{[1 \; 0 \; 1]}_{C} x$$

This system representation is not completely controllable

eigenvalues $\Rightarrow$ $d(s) = (s-1)^2(s-2)$ $\rightarrow$ characteristic equation

$$[\lambda I - A \,\vdots\, B] = \begin{bmatrix} \lambda-1 & -1 & 0 & \vdots & 1 \\ 0 & \lambda-1 & 0 & \vdots & 0 \\ 0 & 0 & \lambda-2 & \vdots & 1 \end{bmatrix}$$

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Week 14

let $\lambda_1 = 1$ (for Hautus test)

$$[\lambda_1 I - A \,\vdots\, B] = \left[\begin{array}{ccc:c} 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 \end{array}\right]$$

$$\begin{array}{ccc} \uparrow & \uparrow & \uparrow \\ c_1 & c_2 & c_3 \end{array}$$

c_1, c_2, c_3 are non-zero vectors

as seen (c_1, c_2, c_3) is not linearly independent, they are linearly dependent.

as $-1c_1 + -1c_2 = c_3$

$$-1\begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} + -1\begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$ (linearly dependent)

** Hence the matrix is rank $= 2 < 3$ (dimention of system representation)

Thus this system representation is not completely controllable as it is not controllable for $\lambda_1 = 1$ mode.

However the system representation is completely observable

$$\begin{bmatrix} \lambda I - A \\ \cdots \\ C \end{bmatrix} = \begin{bmatrix} \lambda-1 & -1 & 0 \\ 0 & \lambda-1 & 0 \\ 0 & 0 & \lambda-2 \\ 0 & 0 & 1 \end{bmatrix}$$

Use Hautus test

For $\lambda_1 = 1$

$$\begin{bmatrix} \lambda_1 I - A \\ \hdashline C \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 1 \end{bmatrix} \begin{array}{l} \rightarrow r_1 \\ \\ \rightarrow r_1 \\ \rightarrow r_3 \end{array}$$

r_1, r_2, r_3 are non-zero vectors

(r_1, r_2, r_3) are linearly independent

Hence the matrix is rank $= 3 = 3$ (dimention of the system representation)

Hence mode corresponding to $\lambda_1 = 1$ is completely observable.

For $\lambda_2 = 2$

$$\begin{bmatrix} \lambda_2 I - A \\ \hdashline C \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{array}{l} \rightarrow r_1 \\ \rightarrow r_2 \\ \\ \rightarrow r_3 \end{array}$$

r_1, r_2, r_3 are non-zero vectors.

(r_1, r_2, r_3) are linearly independant

Hence the matrix is rank $= 3 = 3$ (dimention of the system representation)

Hence mode corresponding to $\lambda_2 = 2$ is completely observable.