To sum-up

modes corresponding to $\lambda_1 = 1$ and $\lambda_2 = 2$ are completely observable, this means the system is completely observable.

Let's find the transfer function of this system:

$$T(s) = C\,(sI - A)^{-1} B = \begin{bmatrix} 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} s-1 & -1 & 0 \\ 0 & s-1 & 0 \\ 0 & 0 & s-2 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

transfer function

Let's find $(sI - A)^{-1}$

$$\begin{bmatrix} s-1 & -1 & 0 \\ 0 & s-1 & 0 \\ 0 & 0 & s-2 \end{bmatrix}^{-1} = \frac{\begin{bmatrix} (1)\begin{vmatrix} s-1 & 0 \\ 0 & s-2 \end{vmatrix} & (-1)\begin{vmatrix} 0 & 0 \\ 0 & s-2 \end{vmatrix} & (1)\begin{vmatrix} 0 & s-1 \\ 0 & 0 \end{vmatrix} \\ (-1)\begin{vmatrix} -1 & 0 \\ 0 & s-2 \end{vmatrix} & (1)\begin{vmatrix} s-1 & 0 \\ 0 & s-2 \end{vmatrix} & (-1)\begin{vmatrix} s-1 & 0 \\ 0 & 0 \end{vmatrix} \\ (1)\begin{vmatrix} -1 & 0 \\ s-1 & 0 \end{vmatrix} & (-1)\begin{vmatrix} s-1 & 0 \\ 0 & 0 \end{vmatrix} & (1)\begin{vmatrix} s-1 & -1 \\ 0 & s-1 \end{vmatrix} \end{bmatrix}^T}{(s-1)^2(s-2)}$$

$$(sI - A)^{-1} = \frac{\begin{bmatrix} (s-1)(s-2) & 0 & 0 \\ (s-2) & (s-1)(s-2) & 0 \\ 0 & 0 & (s-1)^2 \end{bmatrix}^T}{(s-1)^2(s-2)}$$

Week 13



$$(sI-A)^{-1} = \begin{bmatrix} \frac{1}{s-1} & \frac{1}{(s-1)^2} & 0 \\ 0 & \frac{1}{s-1} & 0 \\ 0 & 0 & \frac{1}{s-2} \end{bmatrix}$$

$$T(s) = C\,(sI-A)^{-1}B$$

$$= [1\ 0\ 1] \begin{bmatrix} \frac{1}{s-1} & \frac{1}{(s-1)^2} & 0 \\ 0 & \frac{1}{s-1} & 0 \\ 0 & 0 & \frac{1}{s-2} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$T(s) = [1\ 0\ 1] \begin{bmatrix} \frac{1}{s-1} \\ 0 \\ \frac{1}{s-2} \end{bmatrix} = \frac{1}{s-1} + \frac{1}{s-2} = \frac{s-2+s-1}{(s-1)(s-2)}$$

$$T(s) = \frac{2s-3}{(s-1)(s-2)}$$ → As seen the transfer function is not degree = 3 its degree = 2

Hence there is a pole/zero cancellation in the transfer function as it is not completely controllable (although it is completely observable)

Ex: (4)

$$\dot{x} = \overbrace{\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}}^{A} x + \overbrace{\begin{bmatrix} 1 \\ 1 \end{bmatrix}}^{B} u \qquad y = \overbrace{\begin{bmatrix} 1 & 1 \end{bmatrix}}^{C} x$$

$$Q_C = \begin{bmatrix} B & AB \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$$

controllability matrix

rank($Q_c$) = 2 (full rank) hence completely controllable

$$Q_O = \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$$

observability matrix

rank ($Q_o$) = 2 (full rank) hence completely observable

This system is both observable and controllable hence the transfer function should be $2^{nd}$ order (that is the order of the system representation)

$$T(s) = C\,(sI - A)^{-1} B$$

$$T(s) = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{s-1} & 0 \\ 0 & \frac{1}{s-2} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{s-1} \\ \frac{1}{s-2} \end{bmatrix}$$

$T(s) = \frac{1}{s-1} + \frac{1}{s-2} = \frac{2s-3}{(s-1)(s-2)}$ → order of transfer function is "2"

Ex: (5)

$$\dot{x} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}}_{A} x + \underbrace{\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}}_{B} u \qquad y = \underbrace{\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}}_{C} x$$

$$Q_c = \begin{bmatrix} B & AB & A^2B \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

controllability matrix

$\text{rank}(Q_c) = 1$ (not completely controllable)
(2 modes are uncontrollable)

$$Q_o = \begin{bmatrix} C \\ CA \\ CA^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

observability matrix

$\text{rank}(Q_o) = 1$ (not completely observable)
(2 modes are uncontrollable)

$$T(s) = C(sI-A)^{-1}B = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{s-1} & 0 & 0 \\ 0 & \frac{1}{s-2} & 0 \\ 0 & 0 & \frac{1}{s-3} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$T(s) = \frac{1}{s-1}$ → a first order transfer function for 3rd order system

Week 13



$T(s) = \dfrac{1}{s-1}$ (Since the system is not completely controllable and completely observable the transfer function is of degree $1 \leq 3$ (system degree of representation))